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# A New Venue of Spontaneous Supersymmetry Breaking in Supergravity

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## ABSTRACT

We present a qualitatively new mechanism for dynamical spontaneous breakdown of supersymmetry in supergravity. Specifically, we construct a modified formulation of standard minimal  $N = 1$  supergravity as well as of anti-de Sitter supergravity in terms of a non-Riemannian spacetime volume form (generally covariant integration measure density). The new supergravity formalism naturally triggers the appearance of a *dynamically generated cosmological constant* as an arbitrary integration constant which signifies *spontaneous (dynamical) breaking of supersymmetry*. Applying the new formalism to anti-de Sitter supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a *very small effective observable cosmological constant* as well as a *large physical gravitino mass* as required by modern cosmological scenarios for slowly expanding universe of the present epoch.

## 1. Introduction

Supersymmetry is a fundamental extended space-time symmetry of Nature, believed to manifest itself at ultra-high energies, which is unifying bosons (integer-spin particles) and fermions (half-integer spin particles). Among the principal theoretical highlights of supersymmetry one should mention: drastically reducing the number of a priori independent physical parameters; drastically reducing (in some cases even eliminating) ultra-violet divergencies in quantum field theories; providing possible solution of the hierarchy/fine-tuning problems in high-energy elementary particles phenomenology; naturally appearing within the context of modern string theory. For a very short list of basic references see [1, 2, 3, 4]. Apart from elementary particle physics and cosmology, supersymmetry plays an increasingly active role both as conceptual paradigm and as important

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mathematical tool in various other principal areas of theoretical physics and mathematics such as theoretical condensed matter [5] as well as in the modern theory of integrable (soliton) systems [6].

Unfortunately, supersymmetry is *not an exact* symmetry of Nature. Otherwise, we would observe bosonic (integer-spin) counterparts with the same masses of the fundamental fermionic particles – protons, electrons, *etc.*. Therefore, supersymmetry must be *spontaneously broken* [1, 2].

Under spontaneous symmetry breakdown the symmetry-generating charges in the respective (quantum) field theories are conserved whereas the ground states (“vacuums”) are *not* invariant under the symmetry transformations. Spontaneous symmetry breakdown is always accompanied by the appearance of certain mass (energy) scale of the breakdown. Typically, the scale of spontaneous symmetry breaking is generated by the appearance of non-zero vacuum expectation values of certain (quantum) fields non-trivially transforming under the pertinent symmetry group. The basic example is the *Brout-Englert-Higgs* mechanism in the Standard Model of particle physics (see *e.g.* Ref.[7]).

In minimal supergravity (supersymmetric generalizations of ordinary Einstein general relativity without interactions with other matter fields – for a recent account of modern supergravity theories and notations, see Ref.[4]) there is another way to spontaneously break supersymmetry (supersymmetric Brout-Englert-Higgs effect) – via *dynamical generation of non-zero cosmological constant* [8, 9]. In what follows we describe a new theoretical framework where the above scenario is explicitly realized in a natural way.

The main idea of our current approach comes from Refs.[10] (for recent developments, see Refs.[11]), where some of us have proposed a new class of gravity-matter theories based on the idea that the action integral may contain several terms built with *different* spacetime volume-forms (generally covariant integration measure densities). Namely, apart from terms built with the standard Riemannian volume-form given in terms of the square-root of the determinant of the pertinent Riemannian metric, we may have additional term(s) constructed with one or more [13] *alternative non-Riemannian volume form(s) on the spacetime manifold* in terms of one or more auxiliary antisymmetric tensor gauge field(s) of maximal rank completely independent of the metric. The latter formalism has lead to various new interesting results in all types of known generally coordinate-invariant theories:

- (i)  $D = 4$ -dimensional models of gravity and matter fields containing one term with an alternative non-Riemannian integration measure appear to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, and a natural mechanism for spontaneous breakdown of global Weyl-scale symmetry [10]-[11].
- (ii) Study of reparametrization invariant theories of extended objects (strings and branes) based on employing of a modified non-Riemannian world-sheet/world-volume integration measure [12] leads to dynamically induced variable string/brane tension and to simple string models

of non-abelian confinement.

- (iii) In Refs.[13] a new class of extended gravity-matter models is constructed, built in terms of two independent non-Riemannian volume-forms on the underlying spacetime manifold, producing interesting cosmological implications relating inflationary and today's slowly accelerating phases of the universe.

The principal results described in the next sections are as follows:

(a) First, we briefly outline the main properties of a general class of gravity-matter models with one non-Riemannian volume-form term. Specifically we provide a consistent canonical Hamiltonian analysis of the latter models exhibiting the physical meaning of the pertinent auxiliary fields which are absent in the standard formulation of gravity-matter actions in terms of the ordinary Riemannian spacetime volume-form (see also second Ref.[13]).

(b) The new non-Riemannian volume-form formalism is then applied to minimal  $N = 1$  supergravity in  $D = 4$ -dimensional spacetime. This naturally triggers the appearance of a *dynamically generated* cosmological constant as an arbitrary integration constant, which signifies a new explicit mechanism of spontaneous (dynamical) breaking of supersymmetry.

(c) Applying the same formalism to *anti-de Sitter* supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a very small effective observable cosmological constant as well as a very large physical gravitino mass.

## 2. Gravity-Matter Models With a Non-Riemannian Spacetime Volume-Form

Let us consider the following non-standard gravity-matter system of a general form whose action is a linear combination (with  $c_{1,2}$  – some constants) of one term with an alternative non-Riemannian spacetime volume-form and another one with the standard Riemannian one:

$$S = c_1 \int d^D x \Phi(B) [L^{(1)} + \Phi(H)] + c_2 \int d^D x \sqrt{-g} L^{(2)} \quad (1)$$

Here the following notations are used:

- The alternative volume element in the first term of (1) is given by the following *non-Riemannian* integration measure density:

$$\Phi(B) \equiv \frac{1}{(D-1)!} \varepsilon^{\mu_1 \dots \mu_D} \partial_{\mu_1} B_{\mu_2 \dots \mu_D}, \quad (2)$$

where  $B_{\mu_1 \dots \mu_{D-1}}$  is an auxiliary rank  $(D-1)$  antisymmetric tensor gauge field. The latter can also be parametrized in terms of  $D$  auxiliary scalar fields  $B_{\mu_1 \dots \mu_{D-1}} = \frac{1}{D} \varepsilon_{IJ_1 \dots J_{D-1}} \phi^I \partial_{\mu_1} \phi^{J_1} \dots \partial_{\mu_{D-1}} \phi^{J_{D-1}}$ , so that:  $\Phi(B) = \frac{1}{D!} \varepsilon^{\mu_1 \dots \mu_D} \varepsilon_{I_1 \dots I_D} \partial_{\mu_1} \phi^{I_1} \dots \partial_{\mu_D} \phi^{I_D}$ , but we will stick to the definition (2). The volume element in the second term of (1) is

given by the standard Riemannian integration measure density  $\sqrt{-g}$ , where  $g \equiv \det \|g_{\mu\nu}\|$  is the determinant of the corresponding Riemannian metric  $g_{\mu\nu}$ .

- The Lagrangians  $L^{(1,2)} \equiv \frac{1}{2\kappa^2}R + L_{\text{matter}}^{(1,2)}$  include both standard Einstein-Hilbert gravity action as well as matter/gauge-field parts. Here  $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$  is the scalar curvature within the first-order (Palatini) formalism and  $R_{\mu\nu}(\Gamma)$  is the Ricci tensor in terms of the independent affine connection  $\Gamma_{\lambda\nu}^{\mu}$ .
- In general, the second Lagrangian  $L^{(2)}$  might contain also higher curvature terms like  $R^2$  (see e.g. first Ref.[13]).
- In the first *modified-measure term* of the action (1) we have included an additional term containing the field-strength  $\Phi(H)$  of another auxiliary rank  $(D-1)$  antisymmetric tensor gauge field  $H_{\mu_1\dots\mu_{D-1}}$ :

$$\Phi(H) \equiv \frac{1}{(D-1)!} \varepsilon^{\mu_1\dots\mu_D} \partial_{\mu_1} H_{\mu_2\dots\mu_D}, \quad (3)$$

whose presence is crucial for non-triviality of the model. Such term would be purely topological (total divergence) one if included in standard Riemannian integration measure action like the second term with  $L^{(2)}$  on the r.h.s. of (1).

The auxiliary gauge fields  $B_{\mu_1\dots\mu_{D-1}}$  and  $H_{\mu_1\dots\mu_{D-1}}$  turn out to be pure-gauge (non-propagating) degrees of freedom, however, both are leaving remnants which play crucial role in the sequel (see also next Section). Namely, varying (1) w.r.t.  $H$  and  $B$  tensor gauge fields we get:

$$\partial_{\mu} \left( \frac{\Phi(B)}{\sqrt{-g}} \right) = 0 \quad \rightarrow \quad \frac{\Phi(B)}{\sqrt{-g}} \equiv \chi = \text{const}, \quad (4)$$

$$\partial_{\mu} \left[ L^{(1)} + \Phi(H) \right] = 0 \quad \rightarrow \quad L^{(1)} + \Phi(H) = M = \text{const}, \quad (5)$$

where  $\chi$  (ratio of the two measure densities) and  $M$  are *arbitrary integration constants*. The meaning of  $\chi$  and  $M$  from the point of view of canonical Hamiltonian formalism is elucidated in the next section.

Now, varying (1) w.r.t.  $g^{\mu\nu}$  and taking into account (4)–(5) we arrive at the following effective Einstein equations (in the first-order formalism):

$$R_{\mu\nu}(\Gamma) - \frac{1}{2}g_{\mu\nu}R + \Lambda_{\text{eff}}g_{\mu\nu} = \kappa^2 T_{\mu\nu}^{\text{eff}}, \quad (6)$$

with effective energy-momentum tensor:

$$T_{\mu\nu}^{\text{eff}} = g_{\mu\nu}L_{\text{matter}}^{\text{eff}} - 2 \frac{\partial L_{\text{matter}}^{\text{eff}}}{\partial g^{\mu\nu}}, \quad L_{\text{matter}}^{\text{eff}} \equiv \frac{1}{c_1\chi + c_2} \left[ c_1 L_{\text{matter}}^{(1)} + c_2 L_{\text{matter}}^{(2)} \right], \quad (7)$$



and with a *dynamically generated effective cosmological constant*  $\Lambda_{\text{eff}}$  thanks to the non-zero integration constants  $M, \chi$ :

$$\Lambda_{\text{eff}} = \kappa^2 (c_1 \chi + c_2)^{-1} \chi M . \quad (8)$$

### 3. Canonical Hamiltonian Treatment

In what follows we restrict our attention to  $D = 4$ -dimensional spacetime. For convenience we will introduce the following short-hand notations for the field-strengths (2) and (3) of the auxiliary 3-index antisymmetric gauge fields  $B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$  (the dot indicating time-derivative):

$$\Phi(B) = \dot{B} + \partial_i B^i \quad , \quad B = \frac{1}{3!} \varepsilon^{ijk} B_{ijk} \quad , \quad B^i = -\frac{1}{2} \varepsilon^{ijk} B_{0jk} \quad , \quad (9)$$

$$\Phi(H) = \dot{H} + \partial_i H^i \quad , \quad H = \frac{1}{3!} \varepsilon^{ijk} H_{ijk} \quad , \quad H^i = -\frac{1}{2} \varepsilon^{ijk} H_{0jk} \quad , \quad (10)$$

According to the general form of the action (1) (for simplicity we set here  $c_{1,2} = 1$ ) the pertinent canonically conjugated momenta read:

$$\begin{aligned} \pi_B &= L^{(1)}(u, \dot{u}) + \frac{1}{\sqrt{-g}} (\dot{H} + \partial_i H^i) \quad , \\ \pi_H &= \frac{1}{\sqrt{-g}} (\dot{B} + \partial_i B^i) \quad , \end{aligned} \quad (11)$$

where  $(u, \dot{u})$  collectively denote the set of the basic gravity-matter canonical variables ( $(u) = (g_{\mu\nu}, \text{matter fields})$ ) and their velocities, and:

$$\pi_{B^i} = 0 \quad , \quad \pi_{H^i} = 0 . \quad (12)$$

Eqs.(12) imply that  $B^i, H^i$  will in fact appear as Lagrange multipliers for certain first-class Hamiltonian constraints (see Eqs.(16) below).

Using (11), for the canonical momenta conjugated to the basic gravity-matter canonical variables we obtain:

$$p_u = \pi_H \frac{\partial}{\partial \dot{u}} \left( \sqrt{-g} L^{(1)}(u, \dot{u}) \right) + \frac{\partial}{\partial \dot{u}} \left( \sqrt{-g} L^{(2)}(u, \dot{u}) \right) . \quad (13)$$

Now, from (11) and (13) we obtain the velocities  $\dot{H} = \dot{H}(u, p_u, \pi_H, \pi_B)$ ,  $\dot{B} = \dot{B}(u, \pi_H)$  and  $\dot{u} = \dot{u}(u, p_u, \pi_H, \pi_B)$  as functions of the respective canonically conjugate momenta, wherefrom the canonical Hamiltonian corresponding to (1):

$$\begin{aligned} \mathcal{H} &= p_u \dot{u} + \pi_B \dot{B} + \pi_H \dot{H} \\ &- (\dot{B} + \partial_i B^i) \left[ L^{(1)}(u, \dot{u}) + \frac{1}{\sqrt{-g}} (\dot{H} + \partial_i H^i) \right] - \sqrt{-g} L^{(2)}(u, \dot{u}) \end{aligned} \quad (14)$$

acquires the following form as function of the canonically conjugated variables (here  $\dot{u}=\dot{u}(u, p_u, \pi_H, \pi_B)$ ):

$$\begin{aligned} \mathcal{H} = & p_u \dot{u} - \pi_H \sqrt{-g} L^{(1)}(u, \dot{u}) - \sqrt{-g} L^{(2)}(u, \dot{u}) \\ & + \sqrt{-g} \pi_H \pi_B - \partial_i B^i \pi_B - \partial_i H^i \pi_H . \end{aligned} \quad (15)$$

From (15) we deduce that indeed  $B^i, H^i$  are Lagrange multipliers for the first-class Hamiltonian constraints:

$$\pi_H = \chi = \text{const} \quad , \quad \pi_B = M = \text{const} \quad , \quad (16)$$

which (in virtue of (11)) are the canonical Hamiltonian counterparts of Lagrangian constraint equations of motion (4)-(5).

We conclude that the canonical Hamiltonian treatment of (1) reveals the meaning of the auxiliary 3-index antisymmetric tensor gauge fields  $B_{\mu\nu\lambda}, H_{\mu\nu\lambda}$ . Namely, the canonical momenta  $\pi_B, \pi_H$  conjugated to the “magnetic” parts  $B, H$  (9)-(10) of the respective tensor gauge fields are constrained through Dirac first-class constraints (16) to be constants identified with the arbitrary integration constants  $\chi, M$  (4)-(5) arising within the Lagrangian formulation of the model. The canonical momenta  $\pi_B^i, \pi_H^i$  conjugated to the “electric” parts  $B^i, H^i$  (9)-(10) of the auxiliary 3-index antisymmetric tensor gauge field are vanishing (12) which makes the latter canonical Lagrange multipliers for the above Dirac first-class constraints.

#### 4. Supersymmetric Brout-Englert-Higgs Effect in Minimal Supergravity

Let us now apply the above formalism to construct a non-Riemannian spacetime volume-form version of simplest  $N = 1$  supergravity in  $D = 4$ .

Let us recall the standard component-field action of  $D = 4$  minimal  $N = 1$  supergravity (for definitions and notations we follow [4]):

$$S_{\text{SG}} = \frac{1}{2\kappa^2} \int d^4x e \left[ R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda \right] , \quad (17)$$

$$e = \det \|e_\mu^a\| \quad , \quad R(\omega, e) = e^{a\mu} e^{b\nu} R_{ab\mu\nu}(\omega) . \quad (18)$$

$$R_{ab\mu\nu}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\mu a}^c \omega_{\nu cb} - \omega_{\nu a}^c \omega_{\mu cb} . \quad (19)$$

$$D_\nu \psi_\lambda = \partial_\nu \psi_\lambda + \frac{1}{4} \omega_{\nu ab} \gamma^{ab} \psi_\lambda \quad , \quad \gamma^{\mu\nu\lambda} = e_a^\mu e_b^\nu e_c^\lambda \gamma^{abc} , \quad (20)$$

where all objects belong to the first-order “vierbein” (frame-bundle) formalism.

The vierbeins  $e_\mu^a$  (describing the graviton) and the spin-connection  $\omega_{\mu ab}$  ( $SO(1, 3)$  gauge field acting on the gravitino  $\psi_\mu$ ) are *a priori* independent fields (their relation arises subsequently on-shell);  $\gamma^{ab} \equiv \frac{1}{2} (\gamma^a \gamma^b - \gamma^b \gamma^a)$  *etc.* denote antisymmetrized products of gamma-matrices with  $\gamma^a$  being

the ordinary Dirac gamma-matrices. The invariance of the action (17) under local supersymmetry transformations:

$$\delta_\epsilon e_\mu^a = \frac{1}{2} \bar{\epsilon} \gamma^a \psi_\mu \quad , \quad \delta_\epsilon \psi_\mu = D_\mu \epsilon \quad (21)$$

follows from the invariance of the pertinent Lagrangian density up to a total derivative:

$$\delta_\epsilon \left( e [R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda] \right) = \partial_\mu [e (\bar{\epsilon} \zeta^\mu)] \quad , \quad (22)$$

where  $\zeta^\mu$  functionally depends on the gravitino field  $\psi_\mu$ .

We now propose a modification of (17) by replacing the standard generally-covariant measure density  $e = \sqrt{-g}$  by the alternative measure density  $\Phi(B)$  (Eq.(2) for  $D = 4$ ):

$$\Phi(B) \equiv \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda} \quad , \quad (23)$$

and we will use the general framework described above. The modified supergravity action reads:

$$S_{\text{mSG}} = \frac{1}{2\kappa^2} \int d^4x \Phi(B) \left[ R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3! e} \partial_\mu H_{\nu\kappa\lambda} \right] \quad , \quad (24)$$

where a new term containing the field-strength (Eq.(3) for  $D = 4$ ) of a 3-index antisymmetric tensor gauge field  $H_{\nu\kappa\lambda}$  has been added.

The equations of motion w.r.t.  $H_{\nu\kappa\lambda}$  and  $B_{\nu\kappa\lambda}$  yield:

$$\partial_\mu \left( \frac{\Phi(B)}{e} \right) = 0 \quad \rightarrow \quad \frac{\Phi(B)}{e} \equiv \chi = \text{const} \quad , \quad (25)$$

$$R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3! e} \partial_\mu H_{\nu\kappa\lambda} = 2M \quad , \quad (26)$$

where  $\chi$  and  $M$  are arbitrary integration constants.

The action (24) is invariant under local supersymmetry transformations (21) supplemented by transformation laws for  $H_{\mu\nu\lambda}$  and  $\Phi(B)$ :

$$\delta_\epsilon H_{\mu\nu\lambda} = -e \varepsilon_{\mu\nu\lambda\kappa} (\bar{\epsilon} \zeta^\kappa) \quad , \quad \delta_\epsilon \Phi(B) = \frac{\Phi(B)}{e} \delta_\epsilon e \quad , \quad (27)$$

which algebraically close on-shell, i.e., when Eq.(25) is imposed.

The appearance of the integration constant  $M$  represents a *dynamically generated cosmological constant* in the pertinent gravitational equations of motion and, thus, it signifies a *spontaneous (dynamical) breaking of supersymmetry*.

Indeed, varying (24) w.r.t.  $e_\mu^a$ :

$$\begin{aligned} e^{b\nu} R_{b\mu\nu}^a - \frac{1}{2} \bar{\psi}_\mu \gamma^{a\nu\lambda} D_\nu \psi_\lambda + \frac{1}{2} \bar{\psi}_\nu \gamma^{a\nu\lambda} D_\mu \psi_\lambda \\ + \frac{1}{2} \bar{\psi}_\lambda \gamma^{a\nu\lambda} D_\nu \psi_\mu + \frac{e_\mu^a}{2} \frac{\varepsilon^{\rho\nu\kappa\lambda}}{3! e} \partial_\rho H_{\nu\kappa\lambda} = 0 \end{aligned} \quad (28)$$

and using Eq.(26) (containing the arbitrary integration constant  $M$ ) to replace the last  $H$ -term on the l.h.s. of (28), the results is as follows: we obtain the vierbein counterparts of the Einstein equations including a dynamically generated *floating* cosmological constant term  $e_\mu^a M$ :

$$\begin{aligned} e^{b\nu} R_{b\mu\nu}^a - \frac{1}{2} e_\mu^a R(\omega, e) + e_\mu^a M = \kappa^2 T_\mu^a, \quad (29) \\ \kappa^2 T_\mu^a \equiv \frac{1}{2} \bar{\psi}_\mu \gamma^{a\nu\lambda} D_\nu \psi_\lambda - \frac{1}{2} e_\mu^a \bar{\psi}_\rho \gamma^{\rho\nu\lambda} D_\nu \psi_\lambda - \frac{1}{2} \bar{\psi}_\nu \gamma^{a\nu\lambda} D_\mu \psi_\lambda - \frac{1}{2} \bar{\psi}_\lambda \gamma^{a\nu\lambda} D_\nu \psi_\mu. \end{aligned}$$

Let us recall at this point that according to the classic paper [9] the sole appearance of a cosmological constant in supergravity, even in the absence of a manifest mass term for the gravitino, implies that the gravitino becomes *massive*, i.e., it absorbs the Goldstone fermion of spontaneous supersymmetry breakdown – a *supersymmetric Brout-englert-Higgs effect*.

A significantly more interesting scenario occurs when applying the above formalism with non-Riemannian spacetime volume-forms to anti-de Sitter (AdS) supergravity. Namely, let us start with the standard AdS supergravity action (see e.g. Ref.[4]):

$$\begin{aligned} S_{\text{AdS-SG}} = \frac{1}{2\kappa^2} \int d^4x e \left[ R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda - m \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu - 2\Lambda_0 \right] \quad (30) \\ m \equiv \frac{1}{L}, \quad \Lambda_0 \equiv -\frac{3}{L^2}. \quad (31) \end{aligned}$$

The action (30) contains additional explicit mass term for the gravitino as well as a bare cosmological constant  $\Lambda_0$  balanced in a precise way  $|\Lambda_0| = 3m^2$  so as to maintain local supersymmetry invariance and, in particular, keeping the *physical gravitino mass zero* in spite of the presence of a “bare” gravitino mass term!

At this point let us stress that here we have AdS spacetime as a background (“vacuum”) with curvature radius  $L$  (unlike Minkowski background in the absence of a bare cosmological constant). Therefore, the notions of “mass” and “spin” are now given in terms of the eigenvalues of the Casimirs of the unitary irreducible representations (discrete series) of the group of motion of AdS space  $SO(2, 3) \sim Sp(4, \mathbb{R})$  (for  $D = 4$ ) instead of the ordinary Poincare group  $(SO(1, 3) \times R^4)$  Casimirs (see e.g. Ref.[14]). Thus, on AdS background it is possible for the gravitino to be massive even in the absence of a “bare” gravitino mass term and, *vice-versa*, it will be massless even in the presence of a large “bare” mass provided it is tuned up w.r.t. AdS cosmological constant as in (31).

Now, following the same steps as with (24) we construct a modified AdS supergravity with non-Riemannian spacetime volume element:

$$S_{\text{mod-AdS-SG}} = \frac{1}{2\kappa^2} \int d^4x \Phi(B) \left[ R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda - m \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu - 2\Lambda_0 + \frac{\varepsilon^{\mu\nu\kappa\lambda}}{3!e} \partial_\mu H_{\nu\kappa\lambda} \right], \quad (32)$$

with  $\Phi(B)$  as in (23) and  $m, \Lambda_0$  as in (30). The action (32) is invariant under local supersymmetry transformations:

$$\begin{aligned} \delta_\epsilon e_\mu^a &= \frac{1}{2} \bar{\varepsilon} \gamma^a \psi_\mu \quad , \quad \delta_\epsilon \psi_\mu = \left( D_\mu - \frac{1}{2L} \gamma_\mu \right) \varepsilon \quad , \\ \delta_\epsilon H_{\mu\nu\lambda} &= -e \varepsilon_{\mu\nu\lambda\kappa} (\bar{\varepsilon} \zeta^\kappa) \quad , \quad \delta_\epsilon \Phi(B) = \frac{\Phi(B)}{e} \delta_\epsilon e \quad . \end{aligned} \quad (33)$$

The modified AdS supergravity action (32) will trigger dynamical spontaneous supersymmetry breaking resulting in the appearance of the dynamically generated floating cosmological constant  $M$  as in Eq.(29) which will add to the bare cosmological constant  $\Lambda_0$ . Now we can use the freedom in choosing the value of the *a priori* arbitrary integration constant  $M$  in order to match two basic requirements by modern cosmological scenarios for slowly expanding universe of today [15]. Namely, we can achieve via appropriate choice of  $M \simeq |\Lambda_0| = 3m^2$  a *very small effective observable cosmological constant*:

$$\Lambda_{\text{eff}} = M + \Lambda_0 = M - 3m^2 \ll |\Lambda_0| \quad (34)$$

and, simultaneously, a *large physical gravitino mass*  $m_{\text{eff}}$ :

$$m_{\text{eff}} \simeq m = \sqrt{\frac{1}{3} |\Lambda_0|} \quad , \quad (35)$$

which will be very close to the large “bare” gravitino mass parameter  $m = \sqrt{|\Lambda_0|/3}$  since now because of the smallness of  $\Lambda_{\text{eff}}$  (34) the background spacetime geometry becomes almost flat.

## 5. Conclusions

We have shown that applying the formalism with non-Riemannian spacetime volume-forms in gravity/matter theories provides a simple mechanism for dynamical generation of a cosmological constant. The latter appears as a conserved Dirac-constrained canonical momentum conjugated to the auxiliary maximal-rank antisymmetric gauge field building up the non-Riemannian volume-form. In the context of modified minimal  $N = 1$  supergravity defined in terms of a non-Riemannian spacetime volume-form the dynamically generated cosmological constant triggers spontaneous supersymmetry breakdown and gravitino mass generation (supersymmetric

Brout-Englert-Higgs effect). Upon constructing modified anti-de Sitter supergravity with a non-Riemannian spacetime volume-form we can fine-tune the dynamically generated cosmological integration constant in order to achieve simultaneously a very small physical observable cosmological constant and a very large physical observable gravitino mass – a paradigm of modern cosmological scenarios for slowly expanding universe of the present epoch.

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